Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Tuesday 15 June 2010 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

H35385A

1. (*a*) Show that

$$\frac{\sin 2\theta}{1+\cos 2\theta} = \tan \theta.$$

(2)

(*b*) Hence find, for $-180° \leq \theta < 180°$, all the solutions of

$$\frac{2\sin 2\theta}{1+\cos 2\theta} = 1.$$

Give your answers to 1 decimal place.

(3)

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2. A curve C has equation

$$y = \frac{3}{(5-3x)^2}, \qquad x \neq \frac{5}{3}.$$

The point P on C has x-coordinate 2.

Find an equation of the normal to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

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3. $$\mathrm{f}(x) = 4 \operatorname{cosec} x - 4x + 1, \qquad \text{where } x \text{ is in radians.}$$

(*a*) Show that there is a root α of $\mathrm{f}(x) = 0$ in the interval $[1.2, 1.3]$.

(2)

(*b*) Show that the equation $\mathrm{f}(x) = 0$ can be written in the form

$$x = \frac{1}{\sin x} + \frac{1}{4}$$

(2)

(*c*) Use the iterative formula

$$x_{n+1} = \frac{1}{\sin x_n} + \frac{1}{4}, \qquad x_0 = 1.25,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(*d*) By considering the change of sign of $\mathrm{f}(x)$ in a suitable interval, verify that $\alpha = 1.291$ correct to 3 decimal places.

(2)

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4. The function f is defined by

$$f : x \mapsto |2x - 5|, \quad x \in \mathbb{R}.$$

(*a*) Sketch the graph with equation $y = f(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(2)

(*b*) Solve $f(x) = 15 + x$.

(3)

The function g is defined by

$$g : x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leq x \leq 5.$$

(*c*) Find fg(2).

(2)

(*d*) Find the range of g.

(3)

5.

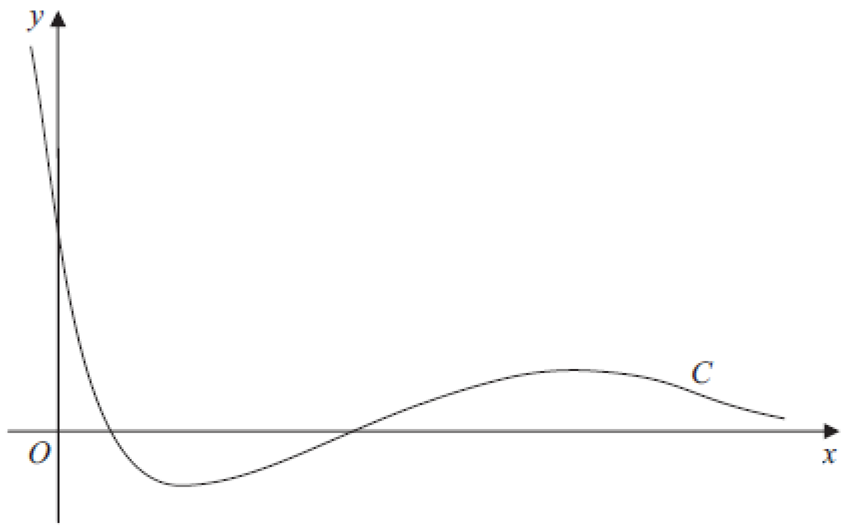

Figure 1

Figure 1 shows a sketch of the curve C with the equation $y = (2x^2 - 5x + 2)e^{-x}$.

(a) Find the coordinates of the point where C crosses the y-axis.

(1)

(b) Show that C crosses the x-axis at $x = 2$ and find the x-coordinate of the other point where C crosses the x-axis.

(3)

(c) Find $\dfrac{dy}{dx}$.

(3)

(d) Hence find the exact coordinates of the turning points of C.

(5)

6.

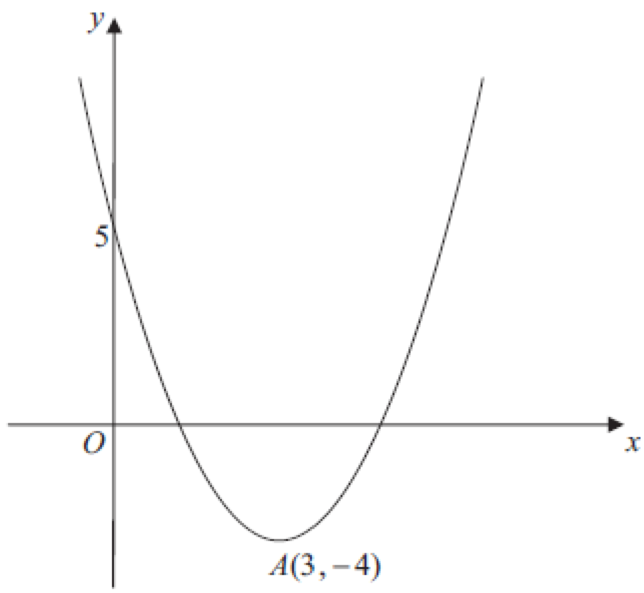

Figure 2

Figure 2 shows a sketch of the curve with the equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve has a turning point at $A(3, -4)$ and also passes through the point (0, 5).

(*a*) Write down the coordinates of the point to which *A* is transformed on the curve with equation

(i) $y = |\mathrm{f}(x)|$,

(ii) $y = 2\mathrm{f}(\frac{1}{2}x)$.

(4)

(*b*) Sketch the curve with equation $y = \mathrm{f}(|x|)$.

On your sketch show the coordinates of all turning points and the coordinates of the point at which the curve cuts the *y*-axis.

(3)

The curve with equation $y = \mathrm{f}(x)$ is a translation of the curve with equation $y = x^2$.

(*c*) Find $\mathrm{f}(x)$.

(2)

(*d*) Explain why the function f does not have an inverse.

(1)

7. (*a*) Express $2 \sin \theta - 1.5 \cos \theta$ in the form $R \sin (\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 4 decimal places.

(3)

(*b*) (i) Find the maximum value of $2 \sin \theta - 1.5 \cos \theta$.

(ii) Find the value of θ, for $0 \leq \theta < \pi$, at which this maximum occurs.

(3)

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2 \sin \left(\frac{4\pi t}{25}\right) - 1.5 \cos \left(\frac{4\pi t}{25}\right), \quad 0 \leq t < 12,$$

where t hours is the number of hours after midday.

(*c*) Calculate the maximum value of H predicted by this model and the value of t, to 2 decimal places, when this maximum occurs.

(3)

(*d*) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

(6)

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8. (*a*) Simplify fully

$$\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}.$$

(3)

Given that

$$\ln (2x^2 + 9x - 5) = 1 + \ln (x^2 + 2x - 15), \quad x \neq -5,$$

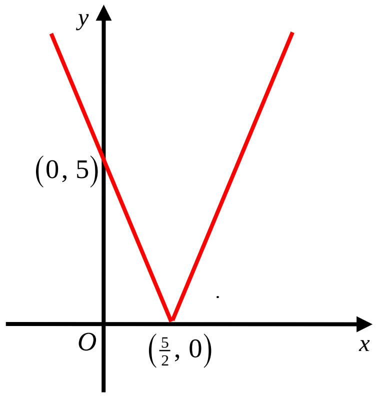
(*b*) find x in terms of e.

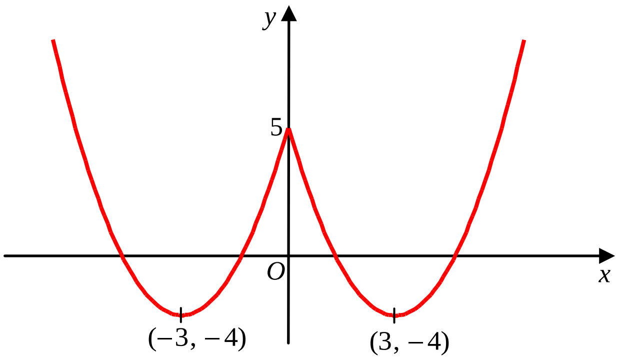
(4)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	$\dfrac{2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$	M1
	$\dfrac{\cancel{2}\sin\theta\cancel{\cos\theta}}{\cancel{2}\cos\theta\cancel{\cos\theta}} = \tan\theta$ (as required) **AG**	A1 **cso**
		(2)
(b)	$2\tan\theta = 1 \Rightarrow \tan\theta = \dfrac{1}{2}$	M1
	$\theta_1 =$ awrt 26.6°	A1
	$\theta_2 =$ awrt -153.4°	A1 $\sqrt{\ }$
		(3)
		[5]
2.	At P, $y = \underline{3}$	B1
	$\dfrac{dy}{dx} = \underline{3(-2)(5-3x)^{-3}(-3)}$ $\left\{\text{or } \dfrac{18}{(5-3x)^3}\right\}$	M1<u>A1</u>
	$\dfrac{dy}{dx} = \dfrac{18}{(5-3(2))^3}$ $\{= -18\}$	M1
	$m(\mathbf{N}) = \dfrac{-1}{-18}$ or $\dfrac{1}{18}$	M1
	N: $y-3 = \frac{1}{18}(x-2)$	M1
	N: $\underline{x-18y+52=0}$	A1
		[7]
3. (a)	$f(1.2) = 0.49166551...$, $f(1.3) = -0.048719817...$	
	Sign change (and as $f(x)$ is continuous) therefore a root α is such that $\alpha \in [1.2, 1.3]$	M1A1
		(2)
(b)	$4\text{cosec}x - 4x + 1 = 0 \Rightarrow 4x = 4\text{cosec}x + 1$	M1
	$\Rightarrow x = \text{cosec}x + \dfrac{1}{4} \Rightarrow \underline{x = \dfrac{1}{\sin x} + \dfrac{1}{4}}$	A1 *
		(2)
(c)	$x_1 = \dfrac{1}{\sin(1.25)} + \dfrac{1}{4}$	M1
	$x_1 = 1.303757858...$, $x_2 = 1.286745793...$	A1
	$x_3 = 1.291744613...$	A1
		(3)
(d)	$f(1.2905) = 0.00044566695...$, $f(1.2915) = -0.00475017278...$	M1
	Sign change (and as $f(x)$ is continuous) therefore a root α is such that	
	$\alpha \in (1.2905, 1.2915) \Rightarrow \alpha = 1.291$ (3 dp)	A1
		(2)
		[9]

Question Number	Scheme	Marks
4. (a)		M1A1
		(2)
(b)	$\underline{x = 20}$	B1
	$2x-5 = -(15+x)$; $\Rightarrow \underline{x = -\frac{10}{3}}$	M1;A1 oe.
		(3)
(c)	$\mathrm{fg}(2) = \mathrm{f}(-3) = \lvert 2(-3)-5\rvert; = \lvert -11\rvert = 11$	M1;A1
		(2)
(d)	$\mathrm{g}(x) = x^2 - 4x + 1 = (x-2)^2 - 4 + 1 = (x-2)^2 - 3$. Hence $\mathrm{g}_{\min} = -3$	M1
	Either $\mathrm{g}_{\min} = -3$ or $\mathrm{g}(x) \geqslant -3$ or $\mathrm{g}(5) = 25 - 20 + 1 = 6$	B1
	$\underline{-3 \leqslant \mathrm{g}(x) \leqslant 6}$ or $\underline{-3 \leqslant y \leqslant 6}$	A1
		(3) **[10]**
5. (a)	Either $y = 2$ or $(0, 2)$	B1
		(1)
(b)	When $x = 2$, $y = (8 - 10 + 2)\mathrm{e}^{-2} = 0\mathrm{e}^{-2} = 0$	B1
	$(2x^2 - 5x + 2) = 0 \Rightarrow (x-2)(2x-1) = 0$	M1
	Either $x = 2$ (for possibly B1 above) or $\underline{x = \frac{1}{2}}$.	A1
		(3)
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} = (4x-5)\mathrm{e}^{-x} - (2x^2 - 5x + 2)\mathrm{e}^{-x}$	M1A1A1
		(3)
(d)	$(4x-5)\mathrm{e}^{-x} - (2x^2 - 5x + 2)\mathrm{e}^{-x} = 0$	M1
	$2x^2 - 9x + 7 = 0 \Rightarrow (2x-7)(x-1) = 0$	M1
	$x = \frac{7}{2}, 1$	A1
	When $x = \frac{7}{2}$, $y = 9\mathrm{e}^{-\frac{7}{2}}$, when $x = 1$, $y = -\mathrm{e}^{-1}$	ddM1A1
		(5) **[12]**

6. (a) (i)	$(3, 4)$	B1 B1
(ii)	$(6, -8)$	B1 B1
		(4)
(b)		B1 B1 B1
		(3)
(c)	$\text{f}(x) = (x-3)^2 - 4$ or $\text{f}(x) = x^2 - 6x + 5$	M1A1
		(2)
(d)	Either: The function f is a many-one {mapping}. Or: The function f is not a one-one {mapping}.	B1
		(1)

7.	(a)	$R = \sqrt{6.25}$ or 2.5 $\tan\alpha = \frac{1.5}{2} = \frac{3}{4} \Rightarrow \alpha = \text{awrt } 0.6435$	B1 M1A1 (3)
	(b) (i) (ii)	Max Value $= 2.5$ $\underline{\sin(\theta - 0.6435) = 1}$ or $\underline{\theta - \text{their}\,\alpha = \frac{\pi}{2}}$; $\Rightarrow \theta = \text{awrt } 2.21$	B1 $\sqrt{}$ <u>M1</u>;A1 $\sqrt{}$ (3)
	(c)	$H_{\text{Max}} = 8.5\ (\text{m})$ $\underline{\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1}$ or $\underline{\frac{4\pi t}{25} = \text{their (b) answer}}$; $\Rightarrow t = \text{awrt } 4.41$	B1 $\sqrt{}$ M1;A1 (3)
	(d)	$\Rightarrow 6 + 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 7$; $\Rightarrow \sin\left(\frac{4\pi t}{25} - 0.6435\right) = \frac{1}{2.5} = 0.4$ $\left\{\frac{4\pi t}{25} - 0.6435\right\} = \sin^{-1}(0.4)$ or awrt 0.41 Either $t = \text{awrt } 2.1$ or awrt 6.7 So, $\left\{\frac{4\pi t}{25} - 0.6435\right\} = \left\{\pi - 0.411517\ldots \text{ or } 2.730076\ldots^{c}\right\}$ Times $= \{14:06,\ 18:43\}$	M1;M1 A1 A1 ddM1 A1 (6) **[15]**
8.	(a)	$\frac{(x+5)(2x-1)}{(x+5)(x-3)} = \frac{(2x-1)}{(x-3)}$	M1 B1 A1 **aef** (3)
	(b)	$\ln\left(\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}\right) = 1$ $\frac{2x^2 + 9x - 5}{x^2 + 2x - 15} = \text{e}$ $\frac{2x - 1}{x - 3} = \text{e} \Rightarrow \quad 3\text{e} - 1 = x(\text{e} - 2)$ $\Rightarrow x = \frac{3\text{e} - 1}{\text{e} - 2}$	M1 dM1 M1 A1 **aef cso** (4) **[7]**